Application of Indefinite Integral and Definite Integral

Zaw Zaw Aung[1]

Abstract

This paper is mainly presented about application of indefinite integral, definite integral and the different concepts between them with real world problems. We construct the mathematical model for integral and approach to application with real world problems. We present the different applications between definite integral and indefinite integral. We also emphasize the application of integral with real world problems.

Keywords: Indefinite Integral, Definite Integral

1. Introduction

Today is technology age. It is very important to be able to solve daily life problems such as business, technology, agriculture and manufacturing and so on. It is essential to know how to apply mathematics, especially, integral calculus in daily life process. There are two kinds of branch in calculus – integral calculus and differential calculus. Both branches can be used to solve different problems. Integral is a branch of mathematics known as integral calculus. Integral calculus involves the essential part of mathematics today. When we know the derivative of a function, we can determine the original function by the use of integral. Integral is mainly used to find the area, volume, average value of a function, surface area, distance, work, velocity and acceleration and so on. In this paper, we present briefly about the application of integral—definite and indefinite integral —with applied examples that involves with business. It is an essential and useful subject in different fields to calculate the estimation of business, economic, military and science and so on. In this paper, the practical applications of definite and indefinite integral are presented with suitable business and economic problems.

2. Objectives of the Study

The objectives of thestudy are:

1. to develop the essential skills in integral calculus with practical applications.
2. to understand the different concepts between indefinite and definite integral.
3. to know how to apply indefinite and definite integral in real world.

3. Indefinite Integral

3.1 Anti-derivatives

A function F is **an anti-derivative** of *f* on an interval *I*if F'(x) = f(x) for all x in *I*.

3.1.1 Theorem 1

Let G be an anti-derivative of a function *f*. Then, every anti-derivative of F of *f* must be of the form *F(x)= G(x)+ C*, where *C* is constant.

1. Assistant Lecturer, Department of Mathematics, Co-operative University, Sagaing

3.2 Indefinite Integral

When the derivative of a given function is known, the process of finding all anti-derivatives of a given function is called **anti-differentiation,** or **integration**. We use the symbol $\int$ **, called an integral sign.** This symbol is an elongated **S**, which is the first letter in the word "**Summation. It** indicates that the operation of integration is to be performed on some functions *f*. The **dx** is called the **differential** and it is to operate the function with respect to x**.** The function *f* is called **integrand**; the constant C is called the **additive constant of integration or a constant of integration**. Finally **anti-derivation** is called **integration**. Thus,

$$\int f(x)dx = F(x) + C, \text{ where } C \text{ is constant.}$$

$\int f(x)dx$ is called the **indefinite integral** of *f*.

3.2.1 Application of Indefinite Integral

It is important to know how to apply the mathematical theory, model and formula in real world. The indefinite integral is a basic theory in integral calculus. Here, the application of the indefinite integral will be presented with real world problems that are mainly used in business and economics.

Suppose the rate of change of a value of a house that cost MMK 30,000,000 can be modelled by

$$\frac{dV}{dt} = 2.4e^{0.08t}$$

Where V is the market value of the house in millions of kyats and $\boldsymbol{t}$ is time in year since the house was purchased. It is to find the function that expresses the predicted value V in term of $\boldsymbol{t}$.To approach the estimated values of the house after two years and ten years.

We start solving the problem with indefinite integral,

$$V = \int \frac{dV}{dt} dt = \int 2.4e^{0.08t} dt$$

$$V = 2.4\int e^{0.08t} (\frac{1}{0.08})(0.08dt)$$

$$V = 30e^{0.08t} + C$$

Using V= 30(million kyats), when t = 0, then the result will become

$$30 = 30 + C$$

C= 0, then the function of time becomes

$$V = 30e^{0.08t}$$

The above function is the original function of the given problem. It gets from the given derivative, $\frac{dV}{dt} = 2.4e^{0.08t}$.

The indefinite integral gives the general function of the problem if the derivative is already known. It can evaluate the different results by substituting the values of variable $\boldsymbol{t}$ in the function.

The predicted value of the house can be estimated by using indefinite integral method. In this problem, we can estimate some values of the house when the time in any year.

First, it can estimate value of that house after two years, t =2. Then, Substitute t =2 as follow

$$V = 30e^{0.08(2)}$$

$$V = 35.9165$$

As the calculation, the house will be worth MMK 35,916,500 after two years. Again, it can solve the value of house after ten years, t =10

$$V = 30e^{0.08(10)}$$

$$V = 66.7662$$

In the results above, the house will be worth MMK 66,766,200 after 10 years. We found that the indefinite integral can estimate the predicted value the original function on different intervals. The indefinite integral not only gives the general function but also provides the predicted value on different intervals if the interval value has already been known.

4. Definite Integral

4.1 Area under the Graph of a Function and Riemann[2] Sum

Let f be a non-negative continuous function on [a, b].Then, the area of the region under the graph of f is

$$A = \lim_{n\to\infty} [f(x_1) + f(x_2) + ... f(x_n)]\Delta x,$$

where $x_1, x_2, ..., x_n$ are arbitrary points in the n subintervals of [a, b] of equal width $\Delta x = (b-a)/n$. $[f(x_1) + f(x_2) + ... f(x_n)]\Delta x$ is called **Riemann Sum**.

4.2 Definite Integral

Let f be a continuous function defined on [a, b]. If

$$\lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + ... f(x_n)\Delta x]$$

exists for all choices of representative points $x_1, x_2, ..., x_n$ in the n subintervals of [a, b] of equal width then this limit of **Riemann Sum** is called the **definite integral** of f from a to b and it is denoted by $\int_a^b f(x)dx$. Thus

$$\int_a^b f(x)dx = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + ... f(x_n)\Delta x]$$

The number ais lower limit of integration and number bis upper limit of integration.

[2]German Mathematician Bernhard Riemann (1826-1866)

4.3 Fundamental Theorem of Calculus

Let f be a continuous function on the interval [a, b], and g be anti-derivatives of f on the interval. The definite integral of f from a to b written $\int_a^b f(x)dx$ is the number g(b)- g(a).

$$\int_a^b f(x)dx = g(b) - g(a)$$

The number **b** is called the **upper limit** of the integral and **a** is called **lower limit**. As with the indefinite integral, $\int f(x)dx$,the function f is called the integrand.

4.4 Application of Definite Integral

Definite integral is the essential field in mathematics. It is important to understand how to apply integral calculus in real world. In this section, we present how to apply it in practical problem solving. We emphasize the Fundamental Theorem of Calculus to solve the definite integral problems.

Suppose that a Vending Machine Service company models its income by assuming that money flows continuously into the machine, with the annual rate of flow given by

$$f(t) = 120e^{0.01t}\ (0 \le \mathrm{t} \le 3)$$

in thousands of kyats per year. It is to find the total income from the machine over the first three years.

In definite integral, we can solve the above problem by using fundamental theorem of calculus. This theorem gives more precise estimated value on the given interval.*I(t)* represents the total income of the company. t represents time in year. First, we start the definite integral with upper and lower limits.

$$I(t) = \int_0^3 f(t)dt$$

$$I(t) = \int_0^3 120e^{0.01t}dt$$

Here, we do calculation as follow

$$I(t) = 120\frac{e^{0.01t}}{0.01}\Big|_0^3$$

$$I(t) = 12000e^{0.01t}\Big|_0^3$$

In this stage, we use the Fundamental Theorem of Calculus by substitution limit values as follow

$$I(t) = 12000e^{0.01(3)} - 12000e^{0.01(0)}$$

$$I(t) = 12365.45 - 1$$

$$I(t) = 12364.45$$

We found that the company's income over the first three years is 12365.45 thousand kyats. Fundamental theorem of calculus helps definite integral to find the exact value between the given intervals. Again, we continue with another problems to understand the application of definite integral.A store finds that its sale revenue change at a rate given by

$$S'(t) = -30t^2 + 360t \quad \text{dollars per day.}$$

where t is the number of day after advertising campaign ends and $0 \leq t \leq 30$.First it is to find the total sale revenue for the first week after the campaign end ($0 \leq t \leq 7$).Next, we will find the total sale revenue for the second week after the campaign end.

The rate of change at time is modelled by the company themselves. The problem solving is as follow:

S(t) represent the total sale in time t.

$$S(t) = \int_0^7 (-30t^2 + 360t)dt$$

$$S(t) = -\frac{30t^3}{3} + \frac{360t^2}{2}\Bigg|_0^7$$

$$S(t) = -10t^3 + 180t^2\Bigg|_0^7$$

$$S(t) = [-10(7)^3 + 180(7)^2] - [-10(0)^3 + 180(0)^2]$$

$$S(t) = 5390$$

As the result, the store earns 5390 dollars for the first week. Then, we solve for the second week after the campaign ends. It means the time in day is from t =7 to t =14. Like the above calculation, we can substitute the interval easily.

$$S(t) = -10t^3 + 180t^2\Bigg|_7^{14}$$

$$S(t) = [-10(14)^3 + 180(14)^2] - [-10(7)^3 + 180(7)^2]$$

$$S(t) = 2450$$

Here, the sale revenue for the second week will be 2450 dollars. Definite integral gives the exact value between the given interval.It is more convenient method in integral calculus. If the rate of change and interval are known, it is easy to find the exact value.

Conclusion

Actually, integral is essential not only for mathematicians but also for business men, engineers and scientists. Indefinite integral describes the general term of the function and it can also give the particular solution when the initial condition is given. It can solve the different solutions in different interval values. The definite integral can be used to find the net value rather than indefinite integral. Like the indefinite integral, the definite integral can be applied in real world. Fundamental theorem of calculus is essential to solve the definite integral .the definite integral is also used in business and economics. The result in real application of both definite and indefinite integral give precise estimated result on each condition. The indefinite integral emphasizes to find the function of the problem and can solve the problems with conditions. Definite integral emphasizes to find the precise value or net value on limited interval. It is concluded that the indefinite integral can give the general function to solve the problem and the definite integral can find the definite result on interval [a, b].

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